AoPS Problem Solving | AoPS Online | Beast Academy | AoPS Academy
AoPS Online
ONLINE SCHOOL BOOKSTORE COMMUNITY RESOURCES
Enroll summer-long with our math, science, competition training, and computer science courses - schedule today!
Students Courses Competitions Programs Academy Resources Parents Support Online Store
Chapter 7: Ratios, Conver...
Review Problems
Search Inside
Review Problems
Review Problems

7.34:

The ratio of cats to dogs at the pound is $2:3$. If there are 99 cats, then how many dogs are there?

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 27

Solution. Multiplying both parts of the ratio by 9 gives

$$\text{cats} : \text{dogs} = 18 : 27,$$

so there are $\boxed{27}$ dogs at the pound.

7.35:

The ratio of boys to girls at a summer camp is 4 to 5. If the total number of students at the camp is 108, then how many boys are at the camp?

Preview: 68

48

Hide Solution | Reset

Your Submission: 48

Solution. Since the ratio of boys to girls at the camp is $4:5$, the boys are $\frac{4}{4+5} = \frac{4}{9}$ of the students at the camp. There are 108 students, so there are $\frac{4}{9} \cdot 108 = \boxed{48}$ boys.

7.36: Source: MATHCOUNTS

Given that one pound is sixteen ounces, what is the ratio of 1 pound, 4 ounces to 3 pounds, 10 ounces?

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 10:29

Solution. We express both weights in ounces, so that we can compare them. 1 pound, 4 ounces equals $1 \cdot 16 + 4 = 20$ ounces. 3 pounds, 10 ounces equals $3 \cdot 16 + 10 = 58$ ounces. So, the ratio of the given weights is $20 : 58$, which is $\boxed{10 : 29}$ in simplest form.

7.37:

A board that is 12 meters long is cut into 2 pieces whose lengths have a ratio of $7 : 5$. What is the length of the longer piece?

Preview: 13

7.5

Hide Solution | Reset

Your Submission: 7.5

Solution. The longer piece is $\frac{7}{7+5} = \frac{7}{12}$ of the whole board, so the length of the longer piece is $\frac{7}{12} \cdot 120$ inches$) = \boxed{70 \text{ inches}}$.

7.38:

The ratio of boys to girls in an assembly is 4 to 3. How many students are present if there are 87 girls?

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 203

Solution 1. The number of girls is $\frac{3}{4+3} = \frac{3}{7}$ of the total number of students, so the total number of students is $\frac{7}{3}$ times the number of girls. Therefore, there are $\frac{7}{3} \cdot 87 = \boxed{203}$ students.

Solution 2. For some value of x, the number of boys is $4x$ and the number of girls is $3x$. Since there are 87 girls, we have $3x = 87$, so $x = \frac{87}{3} = 29$. Therefore, the total number of students is $4x + 3x = 7x = 7(29) = \boxed{203}$.

7.39: Source: AMC8

For every $9!$ Marisa spends, Andie spends $\$2$, Andie spends $\$120$ more than Marisa does. How many dollars does Andie spend?

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 300

Solution. The ratio of the amount Marisa spends to the amount Andie spends is $9 : 5$. So, for some value of x, Marisa spends $5x$ dollars and Andie spends $5x$ dollars. Andie spends $\$120$ more than Marisa spends, so $5x - 2x = 120$. Simplifying gives $3x = 120$, so $x = 40$. Therefore, Andie spends $5 \cdot 60 = \boxed{300 \text{ dollars}}$.

7.40:

Originally, there are 20 fish in a tank, and each fish is a guppy or an angelfish. The ratio of guppies to angelfish in the tank is $3 : 2$. Twenty more fish are added to the tank; each new fish is either a guppy or an angelfish. The ratio of guppies to angelfish after the fish are added is $2 : 1$. How many guppies were added to the tank?

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 6

Solution. Initially, the fraction of fish that are guppies is $\frac{3}{3+2} = \frac{3}{5}$, so there are $\frac{3}{5} \cdot 20 = 12$ guppies in the tank.

After adding the new fish, there are $20 + 20 = 40$ fish in the tank. Since the ratio of guppies to angelfish is then $2 : 1$, we know that $\frac{2}{2+1} = \frac{2}{3}$ of these fish are guppies. So, there are $\frac{2}{3} \cdot 40 = 80$ guppies in the tank, which means $18 - 12 = \boxed{72}$ guppies were added.

7.41: Source: MATHCOUNTS

The four partners in a business decide to split the profits of their company in the ratio $2 : 3 : 3 : 5$. If the profit one year is $\$20,000$, then what is the largest amount of profit received by one of the four partners?

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 10000

Solution. The person who receives the largest amount gets $\frac{5}{2+3+3+5} = \frac{5}{13}$ of the total profit. So, this person receives $\frac{5}{13} \cdot \$20{,}000) = \boxed{\$10{,}000}$.

7.42:

The statue of Abraham Lincoln in the Lincoln Memorial is Washington, D.C., is 6 meters tall. On the back of the $5 bill, the Lincoln Memorial is pictured. The statue of Lincoln is clearly visible. Assuming the back of the $5 bill, and assuming the bill is drawn to scale, then how tall is the actual Memorial in Washington?

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 36 meters

Solution. The statue's height and the Memorial's height are in proportion. On the back of the $5 bill, the Memorial is 25 mm and the statue is 3 mm, so the Memorial is 5 times as tall as the statue. Therefore, the real Memorial is 5 times as tall as the real statue, or $5 \cdot (6 \text{ meters}) = \boxed{30 \text{ meters}}$ tall.

7.43:

On a map, two mountains are $\frac{5}{4}$ inches apart. If $\frac{2}{3}$ of an inch on the map represents 80 miles, then how many miles apart are the two mountains?

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 940

Solution 1. Since $\frac{2}{3}$ inch represents 80 miles, we know that $3 \cdot \frac{1}{2} = 1$ inch represents $2 \cdot 80 = 240$ miles. Therefore $\frac{5}{4}$ inches represents

$$\frac{5}{4} \cdot 240 = \frac{47}{6} \cdot 200 = 47 \cdot 20 = \boxed{940 \text{ miles}}.$$

Solution 2. We use conversion factors. Since $\frac{2}{3}$ inch corresponds to 80 miles, $\frac{5}{4}$ inches corresponds to

$$\begin{aligned} \left(\frac{5}{4} \text{ inches}\right) \frac{80 \text{ miles}}{\frac{2}{3} \text{ inches}} &= \frac{\frac{5}{4} \cdot 80}{\frac{2}{3}} \text{ miles} \\ &= \left(\frac{47}{6} \cdot 80\right) \cdot 3 \text{ miles} \\ &= \boxed{940 \text{ miles}}. \end{aligned}$$

7.44:

A fishing kit is 3 days, and a ride is 8 furlongs. If a desert caravan travels 70 miles per day, then how many furlongs does it travel in a fortnight?

Preview: 1120

1120

Hide Solution | Reset

Your Submission: 1120

Solution. We set up conversion factors:

$$\begin{aligned} 10 \frac{\text{miles}}{\text{day}} &= 10 \frac{\text{miles}}{\text{day}} \cdot \frac{8 \text{ furlongs}}{1 \text{ mile}} \cdot \frac{14 \text{ days}}{1 \text{ fortnight}} \\ &= (10 \cdot 8 \cdot 14) \frac{\text{furlongs}}{\text{fortnight}} \\ &= \boxed{1120} \frac{\text{furlongs}}{\text{fortnight}}. \end{aligned}$$

7.45: Source: MATHCOUNTS

A 5-inch by 5-inch photo is enlarged proportionally such that its smaller dimension is now 1 foot 3 inches. How many inches are in the larger dimension?

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 15in

Solution. The ratio of the shorter dimension to the larger dimension is $3 : 5$. The smaller dimension of the enlarged picture is $12 + 3 = 15$ inches. Multiplying both parts of $3 : 5$ by 5 gives $15 : 25$, so the larger dimension of the picture is $\boxed{25}$ inches.

7.46:

On my tourist map of Québec, the distance between the cities representing Montréal and Québec City is 18 cm. If the scale of the map is $\frac{1}{3}$ cm $= 1$ km, then how far apart are Montréal and Québec City? (Note that $3^3 \text{ cm} = 1 \text{ m}$.)

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 324km

Solution. The ratio of the distance on the map to the actual distance is $\frac{1}{3} \text{ cm} : 1 \text{ km}$. Integers are easier to work with than fractions, so we multiply both parts of the ratio by 3 to give $1 \text{ cm} : 3 \text{ km}$. Next, we notice that the distance on the map between Montreal and Quebec City is in centimeters, not km. In order to use our ratio, we convert the distance on the map to km. Since there are 10 km in every 1 m, 10 cm equals 1 km, so 18 cm equals 180 km. Multiplying both parts of the ratio $1 \text{ cm} : 3 \text{ km}$ by 18 gives an $18 \text{ cm} : 180 \text{ km}$, so Montreal and Quebec City are $\boxed{180 \text{ km}}$ apart.

7.47: Source: MOEMS

Dale travels from city A to city B to city C and back to city A. Each city is 120 miles from the other two; the average rate from city A to city B is 60 mph. Her average rate from city B to city C is 40 mph. Her average rate from city C to city A is 24 mph. What is Dale's average rate for the entire trip, in miles per hour?

Preview: 36 miles/hour

36 miles/hour

Hide Solution | Reset

Your Submission: 36 miles/hour

Solution. Dale's trip from A to B is 120 miles at 60 mph, which takes

$$\frac{120 \text{ miles}}{60 \frac{\text{miles}}{\text{hour}}} = \frac{120}{60} \text{ hours} = 2 \text{ hours}.$$

Her trip from B to C is 120 miles at 40 mph, which takes

$$\frac{120 \text{ miles}}{40 \frac{\text{miles}}{\text{hour}}} = \frac{120}{40} \text{ hours} = 3 \text{ hours}.$$

Her trip from C to A is 120 miles at 24 mph, which takes

$$\frac{120 \text{ miles}}{24 \frac{\text{miles}}{\text{hour}}} = \frac{120}{24} \text{ hours} = 5 \text{ hours}.$$

Combining these, she travels $120 + 120 + 120 = 360$ miles in $2 + 3 + 5 = 10$ hours, so her average speed is

$$\frac{360 \text{ miles}}{10 \text{ hours}} = \frac{360}{10} \frac{\text{miles}}{\text{hour}} = \boxed{36 \frac{\text{miles}}{\text{hour}}}.$$

7.48:

The density of liquid A is 8 pounds per gallon, and the density of liquid B is 6 pounds per gallon. What quantity of liquid B weighs the same as 30 gallons of liquid A?

You may type any additional notes you have here.

Hide Solution | Reset

Your Submission: 40

Solution. 30 gallons of liquid A weighs $30 \cdot 8 = 240$ pounds. Since each gallon of liquid B weighs 6 pounds, we need $240 \text{ lb} = \boxed{40 \text{ gallons}}$ of liquid B to weigh the same amount.

We could also set this up using conversion factors as:

$$\begin{aligned} 30 \text{ gal of } A &= 30 \text{ gal of } A \cdot \frac{8 \text{ lb of } A}{1 \text{ gal of } A} \cdot \frac{1 \text{ lb of } B}{6 \text{ lb of } A} \cdot \frac{1 \text{ gal of } B}{0.75 \text{ lb of } B} \\ &= \frac{30 \cdot 8}{6} \text{ gal of } B \\ &= \boxed{40 \text{ gal}} \text{ of } B. \end{aligned}$$

7.49:

Tess's speedometer (or his motorcycle is broken. He is riding at a constant speed. He times himself and finds that it takes him 1 minute and 20 seconds to ride 1 mile. How fast is he riding in miles per hour?

Preview: division error

division error

Hide Solution | Reset

Your Submission: 32.5mph

Solution. Since 1 minute is 60 seconds, he covers 1 mile in 80 seconds. We can use conversion factors to convert "miles per second" to "miles per hour":

$$\begin{aligned} &\frac{1 \text{ mile}}{80 \text{ seconds}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \\ &= \frac{60 \cdot 60 \text{ mile}}{80 \text{ hour}} \\ &= \boxed{45 \text{ miles per hour}}. \end{aligned}$$

7.50: Source: AMC 8

Elisa swims laps in the pool. When she first started, she completed 10 laps in 25 minutes. Now she can finish 12 laps in 24 minutes. By how many minutes has she improved her lap time?

Preview: half a minute

half a minute

Hide Solution | Reset

Your Submission: half a minute

Solution. When Elisa started, it took her 25 minutes to swim 10 laps, so each lap took her $\frac{25}{10} = \frac{5}{2}$ minutes. Now, she swims 12 laps in 24 minutes, so each lap takes $\frac{24}{12} = 2$ minutes. Since $\frac{5}{2} - 2 = \frac{1}{2}$, she has improved her lap time by $\boxed{\frac{1}{2}}$ minute.

7.51:

A northbound train from Meteor to Jacksonville made the 309-mile journey at an average speed of 61 miles per hour. On its southbound return trip, it made the journey at an average speed of 60 miles per hour. To the nearest tenth of a mile per hour, what was the train's average speed for the 618-mile roundtrip journey?